9/2/16 "Nobody can go back and start a new beginning, but anyone can start today and make a new ending." -Maria Robinson

HW: Finish "Understanding Limits Graphically and Numerically" packet

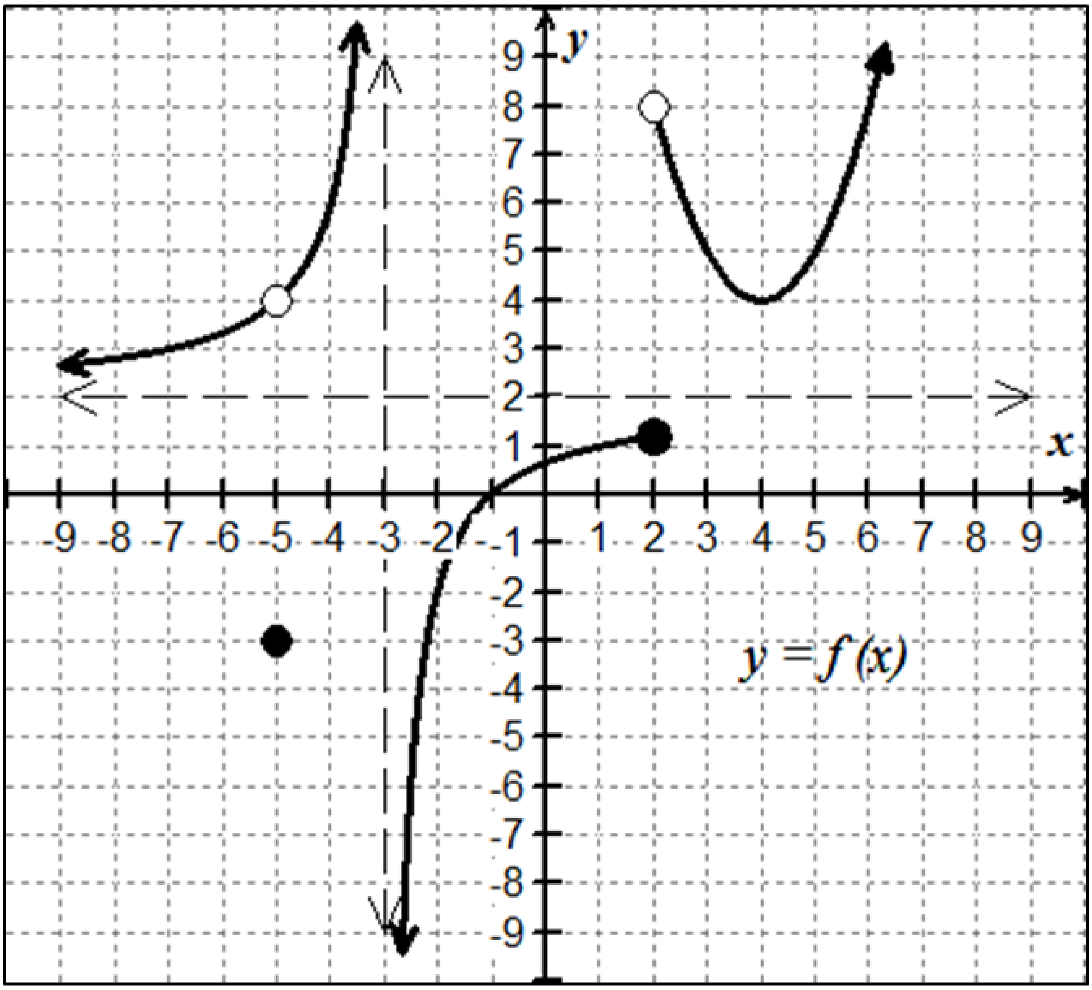
AIM: Understanding Limits Graphically

Warm Up:

Think about "What is a limit?"

Understanding the Limit
Graphically and Numerically

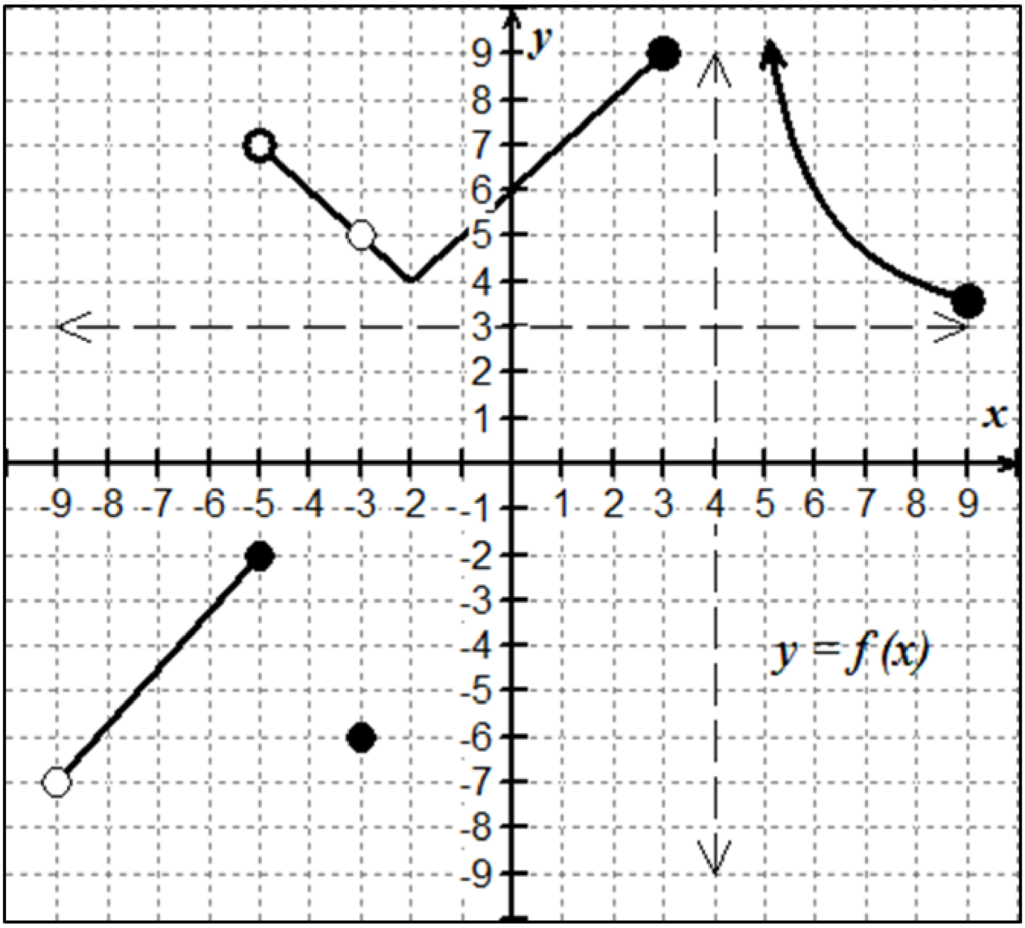
Consider the graph of the function $f(x)$, graphed below:



Using the graph, find the value of each of the following limits.
If a limit does not exist, explain why.

A. $\lim_{x \to -3^-} f(x)$ ∞	B. $\lim_{x \to -5} f(x)$ 4	C. $\lim_{x \to 4} f(x)$ 4
D.) $\lim_{x \to 2^+} f(x)$ 8	E.) $\lim_{x \to 2^-} f(x)$ 1	F.) $\lim_{x \to 2} f(x)$ DNE
G.) $\lim_{x \to -1} f(x)$ 0	H.) $\lim_{x \to -\infty} f(x)$ 2	I.) $\lim_{x \to \infty} f(x)$ ∞

Now you give it a try. Consider the graph shown below to find the value of each of the following limits. If a limit does not exist, explain why.



A. $\lim_{x \to -5^+} f(x)$ 7	B. $\lim_{x \to -2} f(x)$ 4	C. $\lim_{x \to -3} f(x)$ 5
D.) $\lim_{x \to 3^+} f(x)$ DNE (no function as $x \to 3^+$)	E.) $\lim_{x \to 3^-} f(x)$ 9	F.) $\lim_{x \to -5^-} f(x)$ -2
G.) $\lim_{x \to 0} f(x)$ 6	H.) $\lim_{x \to -9} f(x)$ DNE (no function as $x \to -9^-$)	I.) $\lim_{x \to 4^+} f(x)$ ∞

Limits are the "backbone" of understanding that connects algebra and geometry to the mathematics of calculus. In basic terms, a limit is just a statement that tells you what height a function *INTENDS TO REACH* as you get close to a specific *x*-value. Recall from Pre-Calculus that you evaluated three types of limits. Complete the table below:

PROPER LIMIT NOTATIONS		
TYPE OF LIMIT	PROPER NOTATION	VERBALLY:
Right-hand limit	$\lim_{x \to c^+} f(x)$	limit as x approaches c from the right
Left-hand limit	$\lim_{x \to c^-} f(x)$	limit as x approaches c from the left
General limit	$\lim_{x \to c} f(x)$	limit as x approaches c

Consider the function shown below.
Say you want to find $\lim_{x \to 4^+} f(x)$,the positive sign in the limit notation indicates a right-hand limit. If you think of the function as a highway and imagine you are traveling along the graph of $f(x)$ toward $x = 4$ FROM THE RIGHT, NOT TO THE RIGHT, and you stop at the vertical line $x = 4$, the *y*-value where you stop is 3. Therefore, $\lim_{x \to 4^+} f(x) = 3$.

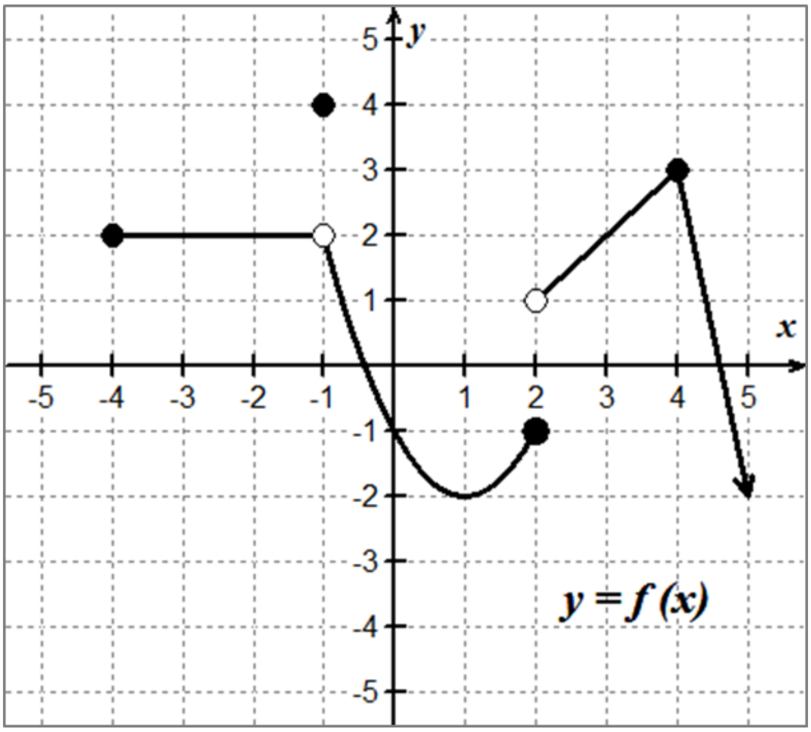


Figure 1-1

You will use this graph to explore the limits for the problems on the next page.

EX #1: Use *Figure 1-1* to find the function values and evaluate each of the following limits:

1. $f(2)$	2. $f(1)$
3. $\lim_{x \to 4^-} f(x)$	4. $\lim_{x \to 2^+} f(x)$
5. $\lim_{x \to 2^-} f(x)$	6. $\lim_{x \to -1^+} f(x)$
7. $\lim_{x \to -1^-} f(x)$	8. $\lim_{x \to -4^+} f(x)$
9. $\lim_{x \to -4^-} f(x)$	10. $\lim_{x \to -1} f(x)$
11. $\lim_{x \to 2} f(x)$	12. $\lim_{x \to 5} f(x)$
13. $\lim_{x \to 0} f(x)$	14. $\lim_{x \to 1} f(x)$

EX #2: THINK ABOUT THIS!

If we think of the function as a highway, then the point at $(2,-1)$ could be considered the end of the road, while the point at $(-1,2)$ is more like a "pothole." How would you describe the points located at

$(2,1)$ dead end without a barrier

$(4,3)$ a bump in road

Hopefully, this analogy gives you a visual reference for understanding limits from a graphical approach. Let's get a little more formal with our definition now.

When finding limits, ask yourself, "What is happening to *y* as *x* gets close to a certain number?" You are finding the ***y*-value** for which the function is approaching as *x* approaches *c*.

LIMIT EXISTENCE THEOREM:

$\lim_{x \to c} f(x)$ exists if and only if

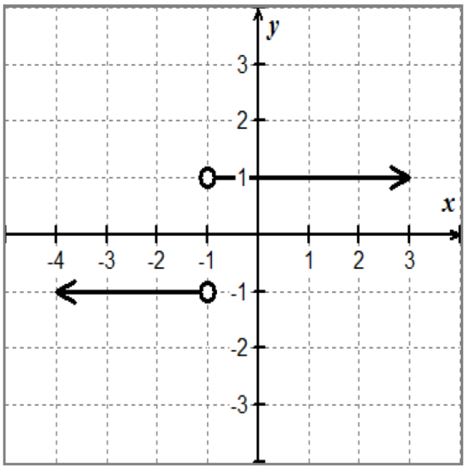
$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$$

L is a real number

Verbally: The limit as *x* approaches *c* on $f(x)$ will exist if and only if the limit as *x* approaches *c* from the left is equal to the limit as *x* approaches *c* from the right.

EX #3: LIMITS CAN FAIL TO EXIST IN THREE SITUATIONS:

CASE 1: limits differ left & right



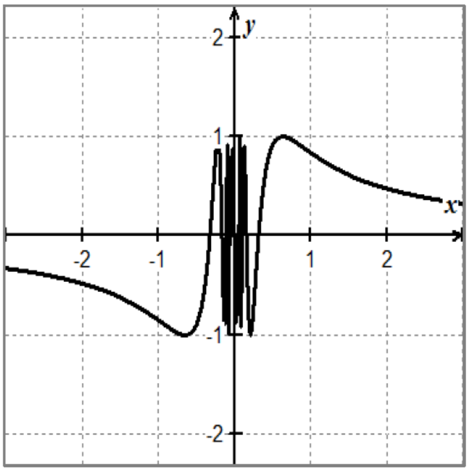
Justify why the limit does not exist at $x = -1$ for $f(x) = \frac{|x+1|}{x+1}$

$\lim_{x \to -1^-} f(x) = -1$

$\lim_{x \to -1^+} f(x) = 1$

Not the same

CASE 2: Oscillating behavior

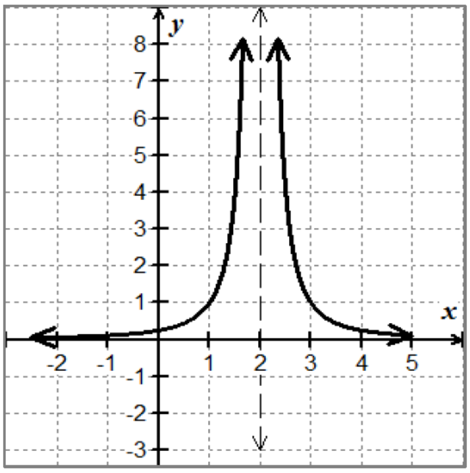


Justify why the limit does not exist at $x = 0$ for $f(x) = \sin\left(\frac{1}{x}\right)$

$\lim_{x \to 0^+} f(x) < 0$

$\lim_{x \to 0^-} f(x) > 0$

CASE 3: limits of ∞ or $-\infty$



Justify why the limit does not exist at $x = 2$ for $f(x) = \frac{1}{(x-2)^2}$

$\lim_{x \to 2^+} f(x) = \infty$

$\lim_{x \to 2^-} f(x) = \infty$

∞ is not real

EX #4: YOU OWN IT! In the box below, complete the sentence in your own words.

In order for the GENERAL LIMIT to exist, the function:

EX #5: Sketch a graph to satisfy each set of conditions.

1. $f(a)$ is undefined 2. $x = a$ is a point discontinuity 3. $\lim_{x \to a} f(x)$ exists	1. $\lim_{x \to a} f(x)$ DNE 2. $x = a$ is a jump discontinuity 3. $f(a)$ is undefined

EX #6: Finding limits from a table of values

Now consider the function $f(x) = \dfrac{x-3}{x^2+2x-15}$.

Complete the table below to find the limit as $x \to 3$.

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$							

Based on your analysis, what are the values of each of the limits below?

$\lim_{x \to 3^-} f(x) =$	$\lim_{x \to 3^+} f(x) =$	$\lim_{x \to 3} f(x) =$